Ein Entscheidungsproblem für positive Polynome (in Dimension 2)

Abstract:
We consider finite sequences $h = (h_1, \dots h_s)$ of real polynomials in $X_1, \dots, X_n$ and assume that the semi-algebraic subset $S(h)$ of $\mathbb{R}^n$ defined by $h_1(a_1, \dots, a_n) \geq 0, \dots, h_s(a_1, \dots, a_n) \geq 0$ is bounded. There exists an abstract valuation theoretic criterion, characterizing those sequences h for which every real polynomial f, strictly positive von $S(h)$, admits a representation

$$f = \sigma_0 + h_1\sigma_1 + \cdots + h_s\sigma_s$$

with each σ_i being a sum of squares of real polynomials. If every h_i is linear, this is always possible. In general, however, it need not be so. We are interested in an effective procedure to decide whether h has this property. Actually, this property is very important in optimization.

In dimension $n = 2$, E. Cabral has given an effective geometric procedure for this decision problem. Recently, S. Wagner has proved decidability for all dimensions using among others model theoretic tools like the Ax-Kochem-Ershov Theorem.