Ein Entscheidungsproblem für positive Polynome (in Dimension 2)

Abstract:
We consider finite sequences \( h = (h_1, \ldots, h_s) \) of real polynomials in \( X_1, \ldots, X_n \) and assume that the semi-algebraic subset \( S(h) \) of \( \mathbb{R}^n \) defined by \( h_1(a_1, \ldots, a_n) \geq 0, \ldots, h_s(a_1, \ldots, a_n) \geq 0 \) is bounded. There exists an abstract valuation theoretic criterion, characterizing those sequences \( h \) for which every real polynomial \( f \), strictly positive von \( S(h) \), admits a representation

\[
f = \sigma_0 + h_1\sigma_1 + \cdots + h_s\sigma_s
\]

with each \( \sigma_i \) being a sum of squares of real polynomials. If every \( h_i \) is linear, this is always possible. In general, however, it need not be so. We are interested in an effective procedure to decide whether \( h \) has this property. Actually, this property is very important in optimization.

In dimension \( n = 2 \), E. Cabral has given an effective geometric procedure for this decision problem. Recently, S. Wagner has proved decidability for all dimensions using among others model theoretic tools like the Ax-Kochem-Ershov Theorem.